4 The Story of Numbers

Number System *A well-defined writing system for expressing numbers using digits or other symbols in a consistent manner, is called a number system.*

Essential Requirements in a Number System Ideally, a number system should fulfil the following requirements:

1. It should contain a set of symbols and digits so as to give every written number a unique representation.
2. It should support or enable an unending standard sequence of numbers that is easy to count with.
3. It should be such that every possible number can be expressed using the defined symbols or digits. There should not be a limitation to the size of the numbers that can be expressed in that system nor should there be a need to devise more symbols to represent larger numbers.

   Here, you will agree that the Hindu numeral system that we use today, comprising digits 0 to 9, fulfils all the above criteria and hence has been well accepted internationally as a standard number system.

Early Counting

As early humans settled down in civilisations during the Stone Age, they felt the need for counting for three basic purposes.

(i) To keep track of prey animals, livestock or stored goods.

(ii) To count members of a tribe living at one place or a smaller group while shifting from one place to another.

(iii) To track time by following up with the cycles of the moon or the change of seasons.

Some of the earliest methods of counting were:

(i) by using fingers or other body parts;

(ii) by using objects like pebbles, sticks or any object available in abundance;

(iii) by making marks or notches on bones or cave walls—an early, crude adaptation of the system of tally marks that we use today in statistics and data keeping.

However, they had their own symbols or numbers and they did not make use of the numbers that we use today.

Evolution of the Hindu Numeral System

The structure of the modern oral and written numbers that we use today originated about 3000 years ago during the Vedic period. Ancient Indian texts such as the *Yajurveda Samhita* mentioned names of numbers based on powers of 10, going up to 10^{12} (i.e., one trillion).

Present day notation	Name in Yajurveda Samhita
1	*eka*
10 (= 10^1)	*dasha*
100 (= 10^2)	*shatha*
1000 (= 10^3)	*sahasra*
10000 (= 10^4)	*ayuta*
10^5	*niyuta*
10^6	*prayuta*

Present day notation	Name in Yajurveda Samhita
10^7	*arbuda*
10^8	*nyarbuda*
10^9	*samudra*
10^{10}	*madhya*
10^{11}	*anta*
10^{12}	*parardha*

The modern number system based on the digits 0–9 was developed in India around 2000 years ago. An early example of this system is found in the *Bakshali* manuscript written around 3rd century CE. This system was a place-value system with a dot to denote the zero, called the *shunyasthana*.

Aryabhata was the first mathematician to fully explain and elaborate scientific computations with the Indian system of 10 symbols around 499 CE.

In 628 CE, astronomer-mathematician Brahmagupta wrote *Brahma-sphuta-siddhanta* which defined zero (*shunya*) as the result of subtracting a number from itself and postulated negative numbers. By the end of 7th century CE decimal numbers began to appear in inscriptions in South-east Asia as well as in India.

Spread to Arab World and Europe

The Indian number system was transmitted to the Arab world by around 800 CE. It gained popularity with its use in the work of the Persian mathematician Al-Khwarizmi titled 'On the Calculation with Hindu Numerals' and the Arab mathematician Al-Kindi who wrote four volumes titled 'On the Use of the Indian Numerals'.

With the translation of Al-Khwarizmi's work into Latin the Hindu numerals were transmitted to Europe. But Fibonacci, an Italian mathematician, is said to have promoted the Indian numerals in Europe with his book '*Liber Abaci*' published in 1202 CE. However, the Roman numerals were so popular in Europe at that time that the Indian numerals did not come into wide use until the invention of printing and the European Renaissance.

However, with the adoption of Indian numerals in Europe, their use spread to all the continents around the world.

Nomenclature

(i) Arab scholars like Al-Khwarizmi and Al-Kindi called the Indian numerals '*Al Hind*' meaning 'from India'.

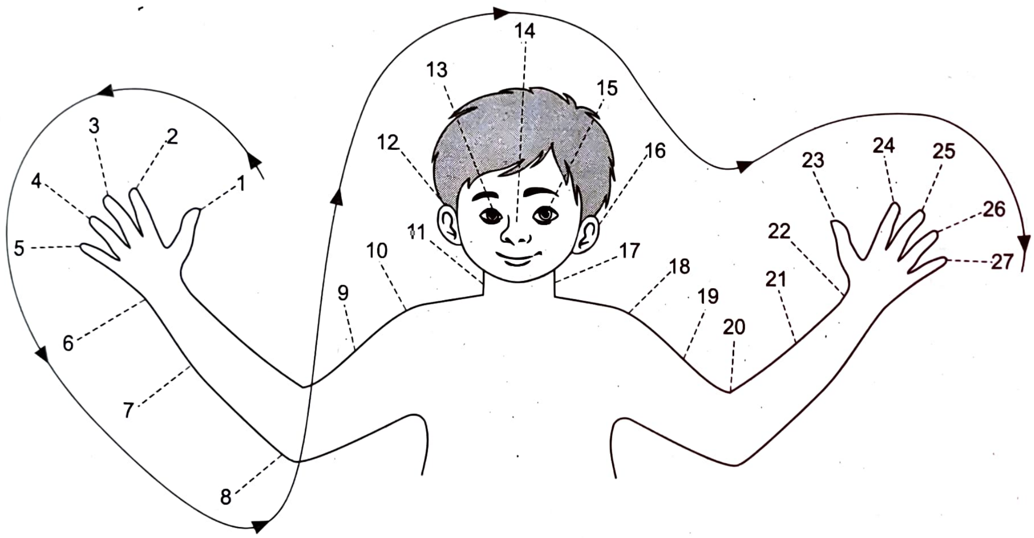
(ii) European scholars called them 'Arabic numerals' as they had learned them from the Arab world.

(iii) As the use of Indian numerals spread to every continent and to every corner of the world, the recent textbooks and documents around the world renamed them as 'Hindu Numerals' or 'Hindu-Arabic Numerals'. Here the word 'Hindu' denoted 'a geography or people from whom these numbers came' and not a religion.

Some Early Number Systems

I. Use of Body Parts

Many groups of people across the world have devised and used number systems using body parts for counting. Such systems use different points on the body to represent numbers, extending beyond the fingers. One such system is the 27-part system used by the Oksapmin people of Papua New Guinea. They begin counting from the thumb of one hand, followed by the fingers in order and move up the arm, neck and head to a halfway point touching specific identified points. They then proceed down the other side of the body in a symmetrical pattern, ending up on the little finger of the other hand, as shown in the diagram below.



II. Tally Marks on Bones and Other Surfaces

Yet another primitive method of counting was by using tally marks. The early tally marks during the prehistoric times were rather crude notches, i.e., marks etched on a surface such as a bone or a wall of a cave. In this method, a mark is made for each object being counted. And the final total number of marks drawn represents the total number of objects.

Archaeologists have unearthed such notched bones dating back more than 20000 years. The oldest known such bones are:

(i) **The Ishango Bone** Found in the Democratic Republic of Congo and dated more than 20000 years ago, it had notches arranged in three columns, possibly indicating calendrical systems.

(ii) **The Lebombo Bone** Discovered in South Africa, this baboon bone is about 44000 years old and has 29 distinct notches, which possibly served as a tally stick or a lunar calendar.

III. Number Names Obtained by Counting in Twos

(i) **Gumulgal System** Gumulgal refers to an indigenous group of people in Australia who are known for their number system, which was based on counting in twos. It was clearly an additive system using the symbol '*urapon*' for one and '*ukasar*' for two. Thus, they wrote number names as shown below.

Number	Name in Gumulgal system	Meaning
1	*urapon*	1
2	*ukasar*	2
3	*ukasar-urapon*	2 + 1
4	*ukasar-ukasar*	2 + 2
5	*ukasar-ukasar-urapon*	2 + 2 + 1
6	*ukasar-ukasar-ukasar*	2 + 2 + 2

Any number greater than 6 was termed '*ras*'. Gumulgal system is considered a significant landmark in the history of numbers because it introduced the idea of counting in a certain group size and using it to represent numbers. This idea laid the foundation of complex number systems that came up later.

(ii) Bushmen of South Africa The Bushmen of South Africa too devised a number system based on using the following symbols.

Symbol	*xa*	*t'oa*	*'quo*	*t'a*
Meaning	1	2	3	increment of 1, i.e., +1

Thus, they represented numbers as shown below.

Number	Representation	Meaning
1	*xa*	1
2	*t'oa*	2
3	*'quo*	3
4	*t'oa-t'oa*	2 + 2
5	*t'oa-t'oa-t'a*	2 + 2 + 1
6	*t'oa-t'oa-t'oa*	2 + 2 + 2

(iii) Bakairi Number System The Bakairis (a group of indigenous people) of South America used an additive system where numbers are formed by combining words for one (*tokale*) and two (*ahage*). Thus, they represented numbers as shown below.

Number	Representation	Meaning
1	*tokale*	1
2	*ahage*	2
3	*ahage-tokale*	2 + 1
4	*ahage-ahage*	2 + 2
5	*ahage-ahage-tokale*	2 + 2 + 1
6	*ahage-ahage-ahage*	2 + 2 + 2

Example 1 *Consider the extension of the Gumulgal number system beyond 6 in the same way of counting by 2s. Come up with ways of performing the different arithmetic operations (+, –, ×, ÷) for numbers occurring in this system, without using Hindu numerals. Use this to evaluate the following:*

(i) (ukasar-ukasar-ukasar-ukasar-urapon) + (ukasar-ukasar-ukasar-urapon)

(ii) (ukasar-ukasar-ukasar-ukasar-urapon) – (ukasar-ukasar-ukasar)

(iii) (ukasar-ukasar-ukasar-ukasar-urapon) × (ukasar-ukasar)

(iv) (ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar) ÷ (ukasar-ukasar)

SOLUTION For ease in calculations, we may use the symbols 'uk' for *ukasar* and 'ur' for *urapon*.

(i) Required sum

$$= (\text{uk-uk-uk-uk-ur}) + (\text{uk-uk-uk-ur})$$
$$= uk + uk + uk + uk + ur + uk + uk + uk + ur$$
$$= uk + uk + uk + uk + uk + uk + uk + ur + ur$$
$$= uk + uk + uk + uk + uk + uk + uk + uk \qquad [\because\ ur + ur = uk]$$
$$= \text{uk-uk-uk-uk-uk-uk-uk-uk}. \qquad [\because\ uk + uk = \text{uk-uk}]$$

Hence, required sum

= *ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar*

(ii) Required difference

$$= (\text{uk-uk-uk-uk-ur}) - (\text{uk-uk-uk})$$
$$= (uk + uk + uk + uk + ur) - (uk + uk + uk)$$
$$= uk + uk + uk + uk + ur - uk - uk - uk$$
$$= uk + ur = \text{uk-ur}.$$

Hence, required difference = *ukasar-urapon*.

(iii) Required product

$$= (\text{uk-uk-uk-uk-ur}) \times (\text{uk-uk})$$
$$= (uk + uk + uk + uk + ur) \times (uk + uk)$$
$$= uk \times (uk + uk + uk + uk + ur) + uk \times (uk + uk + uk + uk + ur)$$
[by distributive law]
$$= uk \times uk + uk \times uk + uk \times uk + uk \times uk + uk \times ur + uk \times uk + uk \times uk + uk \times uk + uk \times uk + uk \times ur$$
$$= \text{uk-uk} + \text{uk-uk} + \text{uk-uk} + \text{uk-uk} + uk + \text{uk-uk} + \text{uk-uk} + \text{uk-uk} + \text{uk-uk} + uk$$
$$[\because\ uk \times uk = \text{uk-uk and } uk \times ur = uk]$$
$$= \text{uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk-uk}.$$

Hence, required product

= *ukasar-ukasar-ukasar* ... 18 *times*.

(iv) Required quotient

$$= (\text{uk-uk-uk-uk-uk-uk-uk-uk}) \div (\text{uk-uk})$$
$$= \frac{(uk + uk + uk + uk + uk + uk + uk + uk)}{(uk + uk)}$$
$$= \frac{(uk + uk)}{(uk + uk)} + \frac{(uk + uk)}{(uk + uk)} + \frac{(uk + uk)}{(uk + uk)} + \frac{(uk + uk)}{(uk + uk)}$$
$$= ur + ur + ur + ur$$
$$= uk + uk \qquad [\because\ ur + ur = uk]$$
$$= \text{uk-uk}.$$

Hence, required quotient = *ukasar-ukasar*.

IV. The Roman Numerals

As the name suggests, this numeral system originated in ancient Rome and remained in prevalence throughout Europe for centuries. In this system, numbers are written with combinations of letters from the Latin alphabet, each denoting a fixed integer value.

The landmark numbers along with their basic symbols, used in Roman numeral system, are as shown below.

Symbol	I	V	X	L	C	D	M
Landmark number	1	5	10	50	100	500	1000

If a bar is placed over a numeral, it is multiplied by 1000.

Thus, $\overline{V} = 5000$ and $\overline{X} = 10000$, etc.

Using these symbols, we may form all Roman numerals by adopting the rules given below.

RULE 1 *Repetition of a symbol in a Roman numeral means addition.*

CAUTIONS (i) Only I, X, C, M can be repeated.

(ii) V, L and D are never repeated.

(iii) No symbol in a Roman numeral can be repeated more than 3 times.

EXAMPLES (i) II = (1 + 1) = 2. (ii) XX = (10 + 10) = 20.

(iii) XXX = (10 + 10 + 10) = 30. (iv) CC = (100 + 100) = 200.

RULE 2 *A smaller numeral written to the right of a larger numeral is always added to the larger numeral.*

EXAMPLES (i) VI = (5 + 1) = 6. (ii) VIII = (5 + 1 + 1 + 1) = 8.

(iii) XV = (10 + 5) = 15. (iv) LX = (50 + 10) = 60.

RULE 3 *A smaller numeral written to the left of a larger numeral is always subtracted from the larger numeral.*

CAUTIONS (i) V, L and D are never subtracted.

(ii) I can be subtracted from V and X only.

(iii) X can be subtracted from L and C only.

(iv) C can be subtracted from D and M only.

EXAMPLES (i) IV = (5 − 1) = 4. (ii) IX = (10 − 1) = 9.

(iii) XL = (50 − 10) = 40. (iv) XC = (100 − 10) = 90.

(v) CD = (500 − 100) = 400. (vi) CM = (1000 − 100) = 900.

RULE 4 *When a smaller numeral is placed between two larger numerals, it is always subtracted from the larger numeral immediately following it.*

EXAMPLES (i) XIV = 10 + (5 − 1) = 14. (ii) XIX = 10 + (10 − 1) = 19.

(iii) CXIV = 100 + 10 + (5 − 1) = 114.

Example 2 *Write Roman numeral for each of the numbers from 1 to 20.*

SOLUTION We may write these numbers as given below.

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

11	12	13	14	15	16	17	18	19	20
XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX

Example 3 *Express each of the following numbers as a Roman numeral.*

(i) 23 (ii) 26 (iii) 29 (iv) 31 (v) 37 (vi) 39 (vii) 40

(viii) 45 (ix) 49 (x) 51 (xi) 63 (xii) 72 (xiii) 79 (xiv) 84

(xv) 89 (xvi) 90 (xvii) 92 (xviii) 99 (xix) 100

SOLUTION We may write these numbers as given below:

(i) 23 = XXIII (ii) 26 = XXVI (iii) 29 = XXIX

(iv) 31 = XXXI (v) 37 = XXXVII (vi) 39 = XXXIX

(vii) 40 = XL (viii) 45 = XLV (ix) 49 = XLIX

(x) 51 = LI (xi) 63 = LXIII (xii) 72 = LXXII

(xiii) 79 = LXXIX (xiv) 84 = LXXXIV (xv) 89 = LXXXIX

(xvi) 90 = XC (xvii) 92 = XCII (xviii) 99 = XCIX

(xix) 100 = C

Example 4 *Represent the following numbers in the Roman system.*

(i) 137 *(ii)* 198 *(iii)* 302 *(iv)* 389 *(v)* 400 *(vi)* 479 *(vii)* 558 *(viii)* 596 *(ix)* 715 *(x)* 1222 *(xi)* 2367 *(xii)* 2999

SOLUTION The given numbers may be represented in the Roman system as under.

(i) $137 = 100 + 30 + 7 =$ CXXXVII.

(ii) $198 = 100 + 90 + 8 =$ CXCVIII.

(iii) $302 = 300 + 2 =$ CCCII.

(iv) $389 = 300 + 80 + 9 =$ CCCLXXXIX.

(v) $400 = 500 - 100 =$ CD.

(vi) $479 = 400 + 70 + 9 =$ CDLXXIX.

(vii) $558 = 500 + 50 + 8 =$ DLVIII.

(viii) $596 = 500 + 90 + 6 =$ DXCVI.

(ix) $715 = 700 + 10 + 5 =$ DCCXV.

(x) $1222 = 1000 + 200 + 20 + 2 =$ MCCXXII.

(xi) $2367 = 2000 + 300 + 60 + 7 =$ MMCCCLXVII.

(xii) $2999 = 2000 + 900 + 90 + 9 =$ MMCMXCIX.

Example 5 *Write each of the following in Hindu-Arabic numeral.*

(i) XXIV *(ii)* XLVI *(iii)* LXXXVI *(iv)* XCIX *(v)* CLXVI *(vi)* CCXXVI *(vii)* CCCXL *(viii)* CDXLVI *(ix)* MCCLXIX *(x)* $M\overline{V}CMXCIX$

SOLUTION We have

(i) XXIV $= 20 + (5 - 1) = 24$.

(ii) XLVI $= (50 - 10) + 5 + 1 = 40 + 6 = 46$.

(iii) LXXXVI $= 50 + 30 + 6 = 86$.

(iv) XCIX $= (100 - 10) + (10 - 1) = 90 + 9 = 99$.

(v) CLXVI $= 100 + 50 + 10 + 5 + 1 = 166$.

(vi) CCXXVI $= 200 + 20 + 5 + 1 = 226$.

(vii) CCCXL $= 300 + (50 - 10) = 300 + 40 = 340$.

(viii) CDXLVI $= (500 - 100) + (50 - 10) + 5 + 1 = 400 + 40 + 6 = 446$.

(ix) MCCLXIX $= 1000 + 200 + 50 + 10 + (10 - 1) = 1000 + 200 + 50 + 10 + 9 = 1269$.

(x) $M\overline{V}CMXCIX = (5000 - 1000) + (1000 - 100) + (100 - 10) + (10 - 1)$

$= 4000 + 900 + 90 + 9 = 4999$.

Example 6 *Show that each of the following is meaningless. Give reason in each case.*

(i) XXXX (ii) VX (iii) IC (iv) XVV

SOLUTION (i) No symbol is repeated more than three times.

∴ XXXX is wrong.

(ii) V, L, D are never subtracted.

∴ VX is wrong.

(iii) I can be subtracted from V and X only.

∴ IC is wrong.

(iv) V, L, D are never repeated.

∴ XVV is wrong.

Example 7 *Add the following numerals without converting them to Hindu numerals.*

(i) CCCXXXII + CCCXIII (ii) LXXXVII + LXXVIII

SOLUTION We have

(i)
$$\begin{array}{ccccc} & \text{D} & \text{XL} & \text{V} & \\ & \boxed{\text{C C C}} & \boxed{\text{X X X}} & \boxed{\text{I I}} & \\ + & \boxed{\text{C C}}\,\boxed{\text{C}} & \boxed{\text{X}} & \boxed{\text{I I I}} & \\ \hline & \text{DC} & \text{XL} & \text{V} & \text{or DCXLV.} \end{array}$$

∴ CCCXXXII + CCCXIII = DCXLV.

(ii)
$$\begin{array}{cccccc} & \text{C} & \text{L} & \text{X} & \text{V} & \\ & \boxed{\text{L}} & \boxed{\text{X X X}} & \boxed{\text{V}} & \boxed{\text{I I}} & \\ + & \boxed{\text{L}} & \boxed{\text{X X}} & \boxed{\text{V}} & \boxed{\text{I I I}} & \\ \hline & \text{C} & \text{L} & \text{X} & \text{V} & \text{or CLXV.} \end{array}$$

∴ LXXXVII + LXXVIII = CLXV.

Multiplication of Roman Numerals

To multiply two Roman numerals without converting them to Hindu numerals, we follow the steps given below.

1. Make a table with 2 columns.
2. Write the two numbers to be multiplied in the first row.
3. Make the next row by halving the first number (discarding remainders) and doubling the second. Continue until there is nothing to halve.
4. Cross out all the rows where the left number is even.
5. Add the remaining numbers in the second column.

The sum so obtained is the product of the two given numbers.

Example 8 *Without converting to Hindu numerals, find the product of the following pairs of landmark numbers.*

(i) V × L (ii) L × D (iii) V × D

SOLUTION We have

(i) V × L

Halving ↓ ↓ Doubling

V	L
~~II~~	~~C~~
I	CC
	CCL ← Product

∴ V × L = CCL.

(ii) L × D

Halving ↓ | Doubling ↓

~~L~~	~~D~~
XXV	M
~~XII~~	~~MM~~
~~VI~~	~~$\overline{\text{IV}}$~~
III	$\overline{\text{VIII}}$
I	$\overline{\text{XVI}}$
	$\overline{\text{XXV}}$ ← Product

∴ L × D = $\overline{\text{XXV}}$.

(iii) V × D

Halving ↓ | Doubling ↓

V	D
~~II~~	~~M~~
I	MM
	MMD ← Product

∴ V × D = MMD.

EXERCISE 4A

1. Express each of the following as a Roman numeral.

(i) 2 (ii) 8 (iii) 14 (iv) 29
(v) 36 (vi) 43 (vii) 54 (viii) 61
(ix) 73 (x) 81 (xi) 91 (xii) 95
(xiii) 99 (xiv) 105 (xv) 114

2. Express each of the following as a Roman numeral.

(i) 164 (ii) 195 (iii) 226 (iv) 341
(v) 475 (vi) 596 (vii) 611 (viii) 759
(ix) 1084 (x) 1333 (xi) 2987 (xii) 3685

3. Write each of the following as a Hindu-Arabic numeral.

(i) XXVII (ii) XXXIV (iii) XLV (iv) LIV
(v) LXXIV (vi) XCI (vii) XCVI (viii) CXI
(ix) CLIV (x) CCXXIV (xi) CCCLXV (xii) CDXIV
(xiii) CDLXIV (xiv) DVI (xv) DCCLXVI

4. Show that each of the following is meaningless. Give reason in each case.

(i) VC (ii) IL (iii) VVII (iv) IXX

HINT (i) V is never subtracted.
(ii) I can be subtracted from V and X only.
(iii) V, L, D are never repeated.
(iv) IX cannot occur to the left of X.

5. Add the following Roman numerals without converting them to Hindu numerals.

(i) XXVIII + LIX
(ii) XCIX + LVIII
(iii) CCLXXXVI + CXLVII
(iv) CCCLXIII + CCCXXIX

6. Find the following products.

(i) VII × IX
(ii) LVI × XXIV
(iii) XXVIII × LXIV
(iv) LIV × LXVI

99 + 58
= 157
CLVII

The Idea of a Base

The Egyptian Number System

The Egyptians developed a written number system around 3000 BCE, using hieroglyphic symbols. They used peculiar landmark numbers to group and represent a given number.

The landmark numbers and the hieroglyphic symbols used in the Egyptian number system are as follows.

Landmark numbers	$1\ (=10^0)$	10^1	10^2	10^3	10^4	10^5	10^6	10^7
Hieroglyphic symbols	𓏺 (line or vertical stroke)	𓎆 (heel bone)	𓍢 (coiled rope)	𓆼 (flower)	𓂭 (bent finger)	𓆐 (tadpole)	𓁨 (god with raised arms)	☼ (sun)

It is clear from the above table that:

(i) the first landmark number is 1 and each next landmark number is 10 times the previous one;

(ii) all the landmark numbers are powers of 10. Such a number system having landmark numbers in which the

(a) first landmark number is 1; and

(b) every next landmark number is obtained by multiplying the current landmark number by some fixed number n, is said to be a base-n number system.

In the Egyptian number system, every next landmark number is obtained by multiplying the current landmark number by 10. So, it is a base-10 system. A base-10 number system is also called a decimal number system.

Representation of Numbers in Egyptian Number System

The Egyptian number system uses a repetitive and additive method to represent numbers. Thus, to represent a number, the appropriate symbol is repeated as many times as needed.

Thus, 3 is written as ||| $(1+1+1)$;

30 is written as ∩∩∩ $(10+10+10)$;

300 is written as 𓍢𓍢𓍢 $(100+100+100)$, and so on.

So, a given number is counted in groups of the landmark numbers, starting from the largest landmark number less than the given number.

For example,

$236 = 200 + 30 + 6$

$= 100 + 100 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 + 1$

Numerals could be written horizontally in any direction (from left to right or right to left) or vertically from top to bottom.

Since symbols are repeated, the value of a symbol always remains the same irrespective of its place in the numeral. So, the Egyptian number system is not a place-value system.

Example 1 *Represent the following numbers in the Egyptian system.*

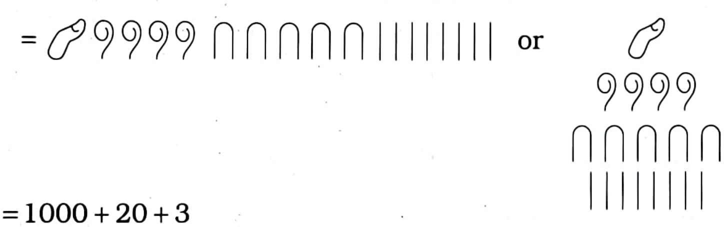
(i) 10458 *(ii)* 1023 *(iii)* 2660 *(iv)* 784

(v) 1111 *(vi)* 70707

SOLUTION We have

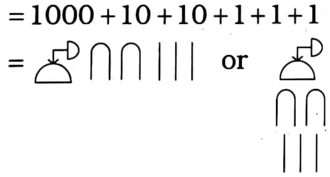
(i) $10458 = 10000 + 400 + 50 + 8$

$= 10000+100+100+100+100+10+10+10+10+10+1+1+1+1+1+1+1+1$



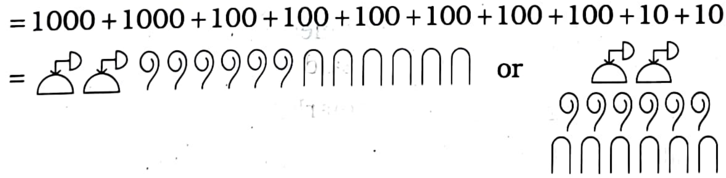
(ii) $1023 = 1000 + 20 + 3$

$= 1000 + 10 + 10 + 1 + 1 + 1$



(iii) $2660 = 2000 + 600 + 60$

$= 1000 + 1000 + 100 + 100 + 100 + 100 + 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10$

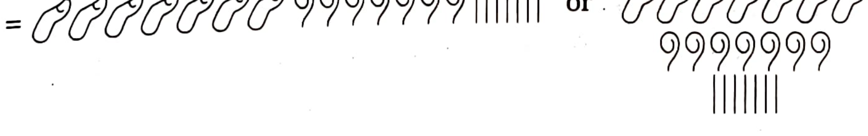


(iv) $784 = 700 + 80 + 4$

$= 100 + 100 + 100 + 100 + 100 + 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$
$+1+1+1+1$

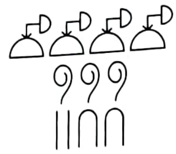
(v) $1111 = 1000 + 100 + 10 + 1 =$

(vi) $70707 = 70000 + 700 + 7$



Example 2 *What numbers do these numerals stand for?*

(i) 

(ii) 

SOLUTION Reading the numerals from top to bottom, we have

(i) Given numeral $= 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 + 1$
$= 200 + 70 + 6 = 276.$

(ii) Given numeral $= 1000 + 1000 + 1000 + 1000 + 100 + 100 + 100 + 10 + 10 + 1 + 1$
$= 4000 + 300 + 20 + 2 = 4322.$

Creating a Base-5 Number System

We have discussed above the basic features of a base-n number system. Using these features we can create a base-5 number system starting with landmark number 1 and then using powers of 5 as subsequent landmark numbers. We may use any symbols to represent these landmark numbers. Thus, we have

Landmark numbers	$1\ (=5^0)$	$5^1\ (=5)$	$5^2\ (=25)$	$5^3\ (=125)$	$5^4\ (=625)$	$5^5\ (=3125)$
Symbols	△	□	⬡	○	∽	↑

Clearly, each landmark number is 5 times the previous one.

Example 3 *Represent the following numbers in the base-5 system created above.*

(i) 143 (ii) 2586 (iii) 10505

SOLUTION Writing the given numerals as the sum of landmark numbers for base-5 system we have

(i) $143 = 125 + 5 + 5 + 5 + 1 + 1 + 1$
= ○□□□△△△

(ii) $2586 = 625 + 625 + 625 + 625 + 25 + 25 + 25 + 5 + 5 + 1$
= ∽∽∽∽⬡⬡⬡□□△

(iii) $10505 = 3125 + 3125 + 3125 + 625 + 125 + 125 + 125 + 125 + 5$
= ↑↑↑∽○○○○□

Example 4 *Is there a number that cannot be represented in our base-5 system above? Why or why not?*

SOLUTION All numbers except 0 can be represented in our base-5 system above. 0 cannot be represented in this system, as the smallest landmark number used in the system is 1.

Example 5 *What are the landmark numbers of a base-n system? Using the above, compute the landmark numbers of a base-7 system.*

SOLUTION The landmark numbers of a base-n number system are the powers of n starting from $n^0 = 1, n, n^2, n^3, n^4, \ldots$.

Clearly, the landmark numbers of a base-7 system are $7^0\ (=1), 7, 7^2, 7^3, 7^4, \ldots$.

Arithmetic Operations in Base-*n* System

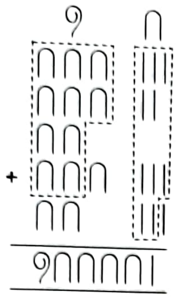
Example 6 *Add the following Egyptian numerals.*

(i) ∩∩∩ ||| and ∩∩∩ |||
∩∩∩ || ∩∩ |||
∩∩

(ii) 𓆼𓆼𓆼 𓍢𓍢𓍢 || and 𓆼𓆼 𓍢𓍢 |||
𓆼𓆼𓆼 𓍢𓍢𓍢 ||| 𓍢𓍢𓍢 ||||
𓆼𓆼𓆼 |||

SOLUTION We have

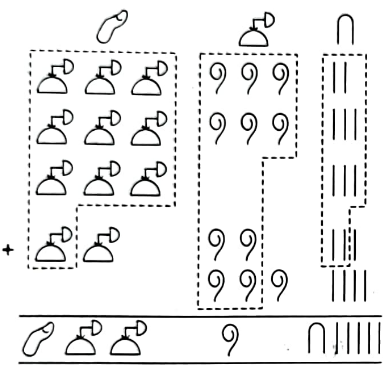
(i)



← Required sum

As shown, 10 | can be regrouped as ∩; and 10 ∩ can be regrouped as 𓍢

(ii)



← Required sum

As shown, 10 | can be regrouped as ∩; 10 𓍢 can be regrouped as 𓆼; 10 𓆼 can be regrouped as 𓂭

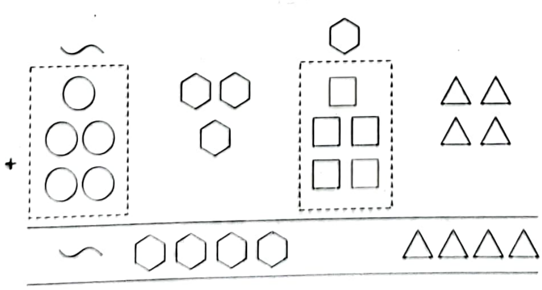
Example 7 *Add the following numerals that are in the base-5 system that we created.*

○⬡⬡□△△ + ○○○○⬡□□□□△△

SOLUTION Remember that in this system, a group of 5 identical symbols will be regrouped to give 1 symbol for the next landmark number as shown below.

Thus, 5 □ can be regrouped into 1 ⬡;

and 5 ○ can be regrouped into 1 ∽.



← Required sum

Example 8 *Find the following products.*

(i) ∩×∩ *(ii)* 𓍢×∩ *(iii)* 𓆼×∩ *(iv)* 𓂭×∩

What do you observe? Write your observation as a general rule for the product of any landmark number with ∩.

SOLUTION We have

(i) 𓎆 × 𓎆 = $10 \times 10 = 10^2$ = 𓍢

(ii) 𓍢 × 𓎆 = $10^2 \times 10 = 10^3$ = 𓆼

(iii) 𓆼 × 𓎆 = $10^3 \times 10 = 10^4$ = 𓂭

(iv) 𓂭 × 𓎆 = $10^4 \times 10 = 10^5$ = 𓆐

We observe that any landmark number multiplied by 𓎆 gives the next landmark number as the product.

Example 9 *Find the following products.*

(i) 𓎆 × 𓆐 (ii) 𓍢 × 𓆼 (iii) 𓆼 × 𓆼 (iv) 𓂭 × 𓆼

What do you conclude?

SOLUTION We have

(i) 𓎆 × 𓆐 = $10 \times 10^5 = 10^6$ = 𓁨

(ii) 𓍢 × 𓆼 = $10^2 \times 10^3 = 10^5$ = 𓆐

(iii) 𓆼 × 𓆼 = $10^3 \times 10^3 = 10^6$ = 𓁨

(iv) 𓂭 × 𓆼 = $10^4 \times 10^3 = 10^7$ = ☼

From the above, we conclude that the product of any two landmark numbers is another landmark number.

Example 10 *Find the following products.*

(i) 𓆼𓎆 × 𓎆 (ii) (𓍢𓍢𓍢𓍢𓍢 𓎆𓎆 ||) × 𓎆

SOLUTION (i) We know that

𓆼𓎆 = 𓆼 + 𓎆

So, applying distributive law, we get

𓆼𓎆 × 𓎆 = (𓆼 + 𓎆) × 𓎆

= 𓆼 × 𓎆 + 𓎆 × 𓎆

$= 10^3 \times 10 + 10 \times 10$

$= 10^4 + 10^2$ = 𓂭𓍢.

(ii) Applying distributive law, we get

required product = 𓍢𓍢𓍢𓍢𓍢 × 𓎆 + 𓎆𓎆 × 𓎆 + || × 𓎆

= 𓆼𓆼𓆼𓆼𓆼 𓍢𓍢 𓎆𓎆

Advantages of Base-*n* System

The base-*n* system makes it easier to perform arithmetic operations on numbers. Thus,

(i) it is easier to add numbers in the base-*n* system because each landmark number has to be grouped by the same size to get the next one.
Thus, in the base-10 system, we know that each landmark number has to be regrouped into 10s to get the next landmark number.

(ii) it is easier to multiply numbers in the base-*n* system because multiplication of landmark numbers gives another landmark number as the product.

Shortcomings of the Egyptian System

The Egyptian system suffered from some major drawbacks, some of which are discussed below.

(i) Drawing symbols is tedious and time-consuming.

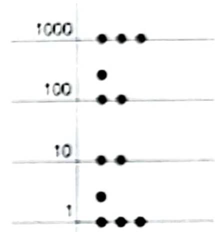
(ii) Since it is not a place value system, symbols have to be drawn repeatedly, e.g., to represent 900, we have to draw 9 times.

(iii) If larger and larger numbers (greater than 10^7) needed to be represented then there was a need for inventing an unending sequence of symbols for higher and higher powers of 10.

Abacus Based on Decimal System

Around 11th century CE, an abacus began to be used as a calculating device. It was constructed on the basis of a decimal system.

It consisted of a board with lines which stood for 1, 10, 100, 1000, etc., from bottom upwards. Thus, starting from the bottom line, each successive line stood for a successive power of 10. A maximum of 4 counters could be marked on a line. The presence of a counter above a line contributed a value of 5.



Let us represent a number say, 3728 on this abacus.

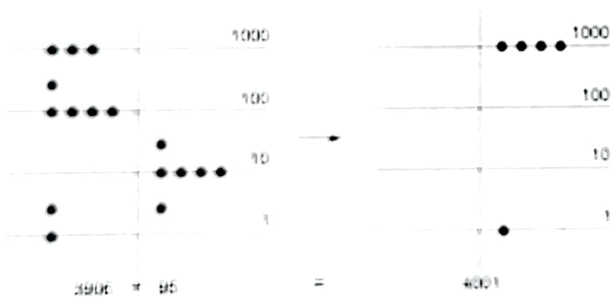
Clearly, we have

$$3728 = 3000 + 700 + 20 + 8$$
$$= 3 \times 1000 + 7 \times 100 + 2 \times 10 + 8 \times 1$$

So, 3728 may be represented on the abacus as shown alongside.

Let us now use the abacus to perform simple addition, say 3906 + 95.

We may represent this addition problem as shown below.



We describe the above working below.

We start with adding the ones and then proceed upwards.

6 ones + 5 ones = 11 ones = 1 ten + 1 one.

So, we place one counter on the ones line.

1 ten (carried) + 9 tens = 10 tens = 1 hundred.

So, we do not place any counter on the tens line.

1 hundred (carried) + 9 hundreds = 10 hundreds = 1 thousand.

So, we do not place any counter on the hundreds line.

1 thousand (carried) + 3 thousands = 4 thousands.

So, we place 4 counters on the thousands line.

Then, 3906 + 95 = 4001.

EXERCISE 4B

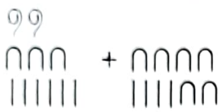
1. Represent the following numbers in the Egyptian system.

(i) 7842 (ii) 6913 (iii) 20202 (iv) 3428

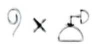
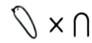
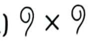
2. What numbers do the following Egyptian numerals stand for?

(i) 
(ii) 
(iii) 

3. Add the following Egyptian numerals.

(i) 
(ii) 

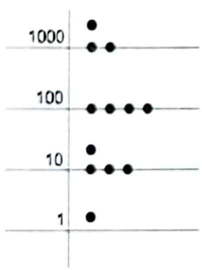
4. Find the following products.

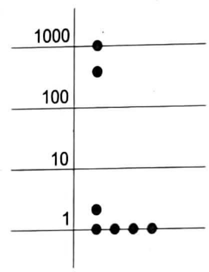
(i) 
(ii) 
(iii) 

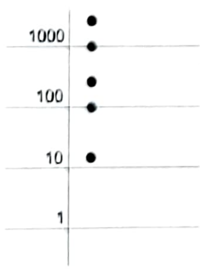
Do the products obtained above share a common property? If yes, what is it?

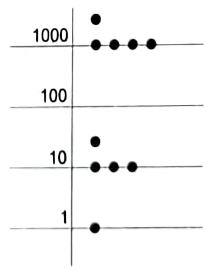
5. Create your own number system of base 4 and represent numbers from 1 to 16.

6. Read the numerals shown on the abacus.

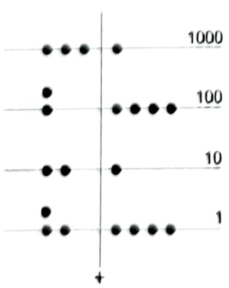
(i)


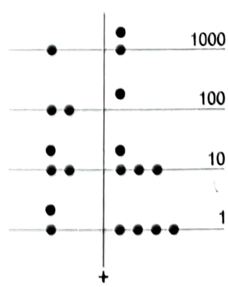
(ii)


(iii)


(iv)


7. Perform the following additions and depict the sum on the abacus.

(i)


(ii)


Place-Value Representation

I. The Mesopotamian Number System

The Mesopotamian number system is known as the first place-value system in the history of numbers.

The Mesopotamians devised a base-60 system, also called the sexagesimal system, for representing numbers.

They may have chosen the base 60 because 60 is a highly composite number with many divisors (1, 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30) which makes it very convenient for calculations involving fractions.

The many divisors of 60 made it suitable for ancient systems of measurement of time and angles, which are prevalent even today.

We know that 1 hour = 60 minutes; 1 minute = 60 seconds.

Likewise, $1° = 60$ minutes (60′); 1 minute (1′) = 60 seconds (60″).

Landmark numbers and symbols in Mesopotamian number system The Mesopotamian sexagesimal system, also known as the Babylonian number system, used powers of 60 as landmark numbers, i.e., 60^0 (= 1), 60, 60^2 (= 3600), 60^3 (= 216000), ..., and so on.

They used the symbol 𒁹 for 1 and 𒌋 for 10.

Using the above two symbols, the numbers from 1 to 59 are represented as shown below.

1	2	3	4	5	6	7	8	9
𒁹	𒈫	𒐈	𒐉	𒐊	𒐋	𒐌	𒐍	𒐎

10	11	12	20	30	40	50	59
𒌋	𒌋𒁹 (10 + 1)	𒌋𒈫 (10 + 2)	𒎙	𒌍	𒐏	𒐐	𒐐𒐎 (50 + 9)

Thus, in the above number system,

26 will be written as 𒎙𒐋,

37 will be written as 𒌍𒐌, and so on.

Let us now represent bigger numbers in the Mesopotamian number system.

Example 1 *Represent the following numbers in the Mesopotamian system.*

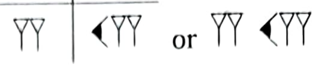
(*i*) 63 (*ii*) 132 (*iii*) 200 (*iv*) 640 (*v*) 7530

SOLUTION We may represent the given numbers as shown below.

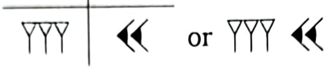
(i) $63 = 1 \times 60 + 3$

Landmark numbers (place value)	60	1	
Number	𒁹	𒐈	or 𒁹 𒐈

(ii) $132 = 2 \times 60 + 12$

Landmark numbers (place value)	60	1
Number		

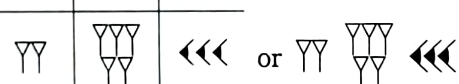
(iii) $200 = 3 \times 60 + 20$

Landmark numbers (place value)	60	1
Number		

(iv) $640 = 10 \times 60 + 40$

Landmark numbers (place value)	60	1
Number		

(v) $7530 = 2 \times 3600 + 5 \times 60 + 30$

Landmark numbers (place value)	3600	60	1
Number			

You must have noticed in the above examples that in the Mesopotamian number syste the rightmost set of symbols showed the number of 1s, the set of symbols to its l showed the number of 60s, the next showed the number of 3600s, and so on. Whenev there was no occurrence of a power of 60, a blank space was given in that position.

Such a number system that makes use of the position of each symbol in determini the landmark number that it is associated with is called a positional number system a place-value system.

Thus, the same symbol placed at different positions in the number will repres different values as shown below.

Landmark numbers (place value)	3600	60	1
Number			

$3600 + 60 + 1 = 3661$.

Landmark numbers (place value)	3600	60	1	
Number	𒌋	𒌋	𒌋	or 𒌋𒌋𒌋
	↓	↓	↓	
	10×3600	10×60	10×1	
	↓	↓	↓	

$$36000 + 600 + 1 = 36610.$$

Advantage of a Place-Value System

As shown above, it is clear that the place-value system helps to represent an unending sequence of numbers using only a finite number of different symbols. Thus, we could represent any number using only two symbols in the above examples.

Defects in the Mesopotamian Number System

The Mesopotamian number system cannot be considered as a fully developed place-value system as it has certain defects which can lead to confusion while reading a number. Let us represent the following numerals in the Mesopotamian number system.

(i) 1

Landmark numbers (place value)	60	1	
Number		𒁹	or 𒁹

(ii) 60

Landmark numbers (place value)	60	1	
Number	𒁹		or 𒁹

(iii) 3600

Landmark numbers (place value)	3600	60	1	
Number	𒁹			or 𒁹

Clearly, the representation of 1, 60 and 3600 is the same except for the difference in the position of the symbol in the place-value chart. This happens because there is no concept or symbol for zero in this number system and empty gaps are left for non-existing values.

Again, let us represent the following numerals in the Mesopotamian number system.

(i) 12

Landmark numbers (place value)	60	1	
Number		𒌋𒁹𒁹	or 𒌋𒁹𒁹
		↓	
		$10 + 2 = 12.$	

(ii) 602

Landmark numbers (place value)	60	1
Number	𒌋	𒐖 or 𒌋𒐖

↓

$10 \times 60 + 2 \times 1 = 602.$

(iii) 36002

Landmark numbers (place value)	3600	60	1
Number	𒌋		𒐖 or 𒌋 𒐖

↓

$10 \times 3600 + 2 \times 1 = 36002.$

Clearly, the representation of 12, 602 and 36002 is the same except for the difference in the spaces between the symbols. This creates confusion as one may read the representation of 602 as 12 and that of 36002 as 602.

The concept of blank space is ill-defined and can be misinterpreted. To overcome this ambiguity, the later Mesopotamians assigned a 'placeholder symbol'  to denote a blank space. This symbol does not have any value and denotes nothingness, just like 0 in the Hindu number system. But this symbol too was used in the middle of numbers and not at the end.

Thus, 12 could be written as 𒌋𒐖;

602 could be written as 𒌋𒐖;

36002 could be written as 𒌋  𒐖.

So, the anomaly in reading 12 and 602 still remains.

Also, 1, 60 and 3600 would still be represented as 𒁹, which is also confusing.

II. The Mayan Number System

The Mayan civilisation flourished in Central America between the 3rd and 10th centuries CE. The Mayans devised a number system which was almost a base-20 system. This number system holds an important position in the advancement of numbers because of

(i) the place-value notation,

(ii) the introduction of a placeholder symbol for the modern-day '0' (zero).

You must recall here that the lack of such a symbol posed great problems in reading numerals in the Mesopotamian number system.

Characteristics of the Mayan Number System

(i) The landmark numbers in the Mayan system were

$1\ (= 20^0), 20\ (= 20 \times 1), 360\ (= 20 \times 18), 7200\ (= 20^2 \times 18), 144000\ (= 20^3 \times 18), \ldots$, and so on.

Thus, the Mayan system deviated slightly from a pure base-20 system as $360\ (= 20 \times 18)$ was taken instead of $400\ (= 20 \times 20)$. This was done to make it compatible for calendar purposes. The subsequent place values are obtained by multiplying by 20 at each step.

(ii) The system used three symbols in all namely.

(a) Dot • representing one (1);

(b) Bar —— representing five (5); and

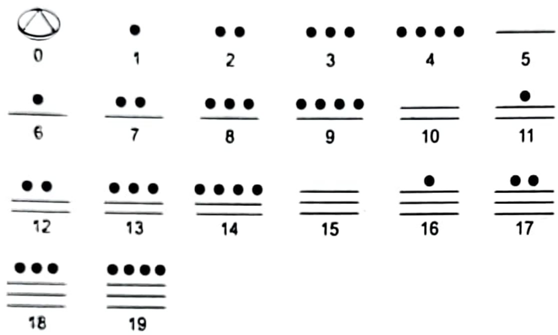
(c) Seashell ◬ representing zero (0).

(iii) Numbers were written vertically, with the largest place value at the top and the smallest place value at the bottom.

(iv) For a given place, there could be a maximum of three bars and four dots.

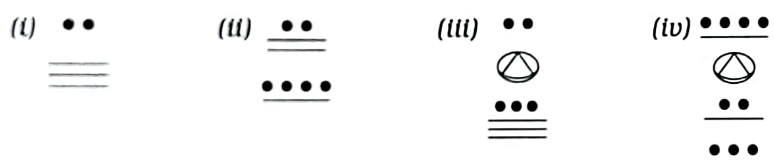
Thus, a group of 5 dots would be represented by one bar and a group of four bars (= 20) would become one dot in the next place up.

Numbers from 1 to 19 in the Mayan number system



Let us now read and represent some larger numbers in the Mayan number system.

Example 2 *What does each of the following numerals represent in the Mayan number system?*



SOLUTION We may arrange the given numerals in the Mayan place-value chart as shown below.

	(i)		(ii)		(iii)		(iv)	
7200							••••	9
360					••	2	◬	0
20	••	2	•• — —	12	◬	0	•• —	7
1	— — —	15	••••	9	••• — — —	18	•••	3
Landmark numbers (place values)	Symbols	Meaning of symbols	Symbols	Meaning of symbols	Symbols	Meaning of symbols	Symbols	Meaning of symbols

Thus, the given numerals may be read as below.

(i) Given numeral $= 2 \times 20 + 15 \times 1 = 40 + 15 = 55$.

(ii) Given numeral $= 12 \times 20 + 9 \times 1 = 240 + 9 = 249$.

(iii) Given numeral $= 2 \times 360 + 0 \times 20 + 18 \times 1 = 720 + 0 + 18 = 738$.

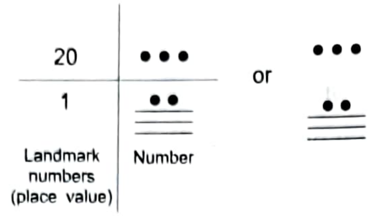
(iv) Given numeral $= 9 \times 7200 + 0 \times 360 + 7 \times 20 + 3 \times 1 = 64800 + 0 + 140 + 3 = 64943$.

Example 3 *Represent each of the following numbers using the Mayan system.*

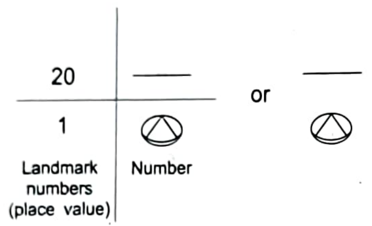
(*i*) 77 (*ii*) 100 (*iii*) 361 (*iv*) 761 (*v*) 1126 (*vi*) 20504

SOLUTION We have

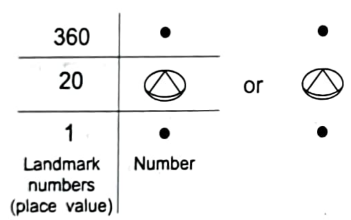
(i) $77 = 3 \times 20 + 17 \times 1$.



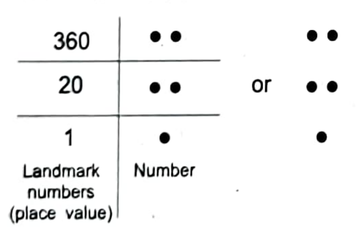
(ii) $100 = 5 \times 20 + 0 \times 1$.



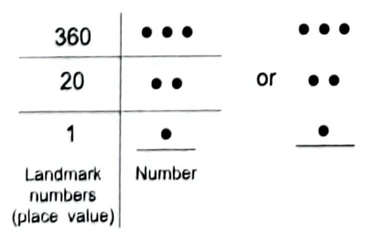
(iii) $361 = 1 \times 360 + 0 \times 20 + 1 \times 1$.



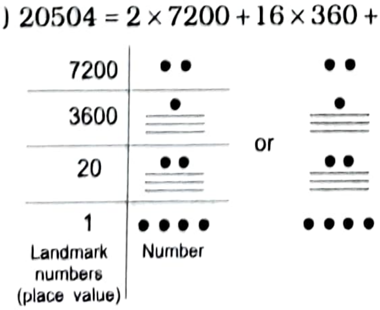
(iv) $761 = 2 \times 360 + 2 \times 20 + 1 \times 1$.



(v) $1126 = 3 \times 360 + 2 \times 20 + 6 \times 1$.



(vi) $20504 = 2 \times 7200 + 16 \times 360 + 17 \times 20 + 4 \times 1$.



III. The Chinese Number System

The Chinese used two number systems—a written system for recording quantities, and a system making use of rods for performing calculations. The numerals in the rod-based number system are called rod numerals. The rod numerals were devised by the Chinese around 3rd century CE and were used until the 17th century.

The system of rod numerals was a true positional numeral (or place value) system with symbols for 1 to 9 and a blank for 0. Clearly, it is a base-10 or decimal number system.

Characteristics of Rod Numerals

(i) They consisted of two different sets of symbols for digits 1 to 9, called the *Zongs* and the *Hengs*. One set of symbols, the *Zongs*, consisted of vertical bars and were used for odd place values (units, hundreds, ten thousands, etc.)

The other set of symbols, the *Hengs*, consisted of horizontal bars and were used for even place values (tens, thousands, etc.)

Numeral	1	2	3	4	5	6	7	8	9
Zongs	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Hengs	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

(ii) The system is a base-10 system. So, the landmark numbers or place-value positions are $1\ (=10^0)$, $10\ (=10^1)$, $100\ (=10^2)$, $1000\ (=10^3)$, and so on.

(iii) Similar to the Mesopotamian system, the rod numerals lacked a symbol for zero and used a blank space to indicate a missing place value. However, as the symbols used for 1 to 9 were of regular shape and uniform size, the blank spaces were easier to locate as compared to the Mesopotamian system.

Let us now read and represent some numbers in the form of rod numerals.

Example 4 *What does each of the following numerals stand for?*

(i) 𝍮𝍦 *(ii)* 𝍢𝍱𝍠 *(iii)* 𝍯𝍡𝍬𝍤 *(iv)* 𝍪𝍥𝍫𝍣

SOLUTION We first write the given numerals in the place-value chart for rod numerals as shown below.

Landmark numbers (place values) →		10^3 (Heng)	10^2 (Zong)	10 (Heng)	1 (Zong)
(i)	Digits			𝍮	𝍦
	Meaning			6	7
(ii)	Digits		𝍢	𝍱	𝍠
	Meaning		3	9	1
(iii)	Digits	𝍯	𝍡	𝍬	𝍤
	Meaning	7	2	4	5
(iv)	Digits	𝍪	𝍥	𝍫	𝍣
	Meaning	2	6	3	4

Thus, we have

(i) given numeral $= 6 \times 10 + 7 \times 1 = 60 + 7 = 67$.

(ii) given numeral $= 3 \times 10^2 + 9 \times 10 + 1 \times 1 = 300 + 90 + 1 = 391$.

(iii) given numeral $= 7 \times 10^3 + 2 \times 10^2 + 4 \times 10 + 5 \times 1 = 7000 + 200 + 40 + 5 = 7245$.

(iv) given numeral $= 2 \times 10^3 + 6 \times 10^2 + 3 \times 10 + 4 \times 1 = 2000 + 600 + 30 + 4 = 2634$.

Example 5 *Represent each of the following numbers in the form of rod numerals.*

(i) 85 (ii) 461 (iii) 5796 (iv) 8328

SOLUTION Clearly, we have

	Number	Representation using rod numerals
(i)	85	𝍰𝍤
(ii)	461	𝍣𝍮𝍠
(iii)	5796	𝍭𝍦𝍱𝍥
(iv)	8328	𝍰𝍢𝍪𝍧

Example 6 *Why do you think the Chinese alternated between the Zong and Heng symbols? If only the Zong symbols were to be used, how would 41 be represented? Could this numeral be interpreted in any other way if there is no significant space between two successive positions?*

SOLUTION The Chinese alternated between the *Zong* and *Heng* symbols so that all the place values stood out distinctly from each other. This was done to avoid any sort of misinterpretation or confusion while reading the numerals, e.g., if only the *Zong* symbols were to be used, the numeral 41 would be written as |||| |. Now, this representation could have been read as 5 if there were no significant space between the units and tens positions or this space was somehow neglected.

The Hindu Number System

The Hindu number system, also called the Indian number system or Hindu-Arabic numeral system, is a base-10 or decimal system which originated in India between the 1st and 4th centuries CE. It is now used as a standard number system all over the world. It uses ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to represent numbers and the landmark number positions or standard place values are $1\ (=10^0), 10\ (=10^1), 100\ (=10^2), \ldots,$ and so on.

The use of 0 as a digit in this system helped to do away with any kind of ambiguity which we came across in the earlier number systems. The invention of zero allowed for the development of a true place-value system as it acted as a placeholder to distinguish numbers like 3 and 30.

Thus, a number say, 7965 can be represented in Hindu number system as below.

$$7965 = 7000 + 900 + 60 + 5 = 7 \times 10^3 + 9 \times 10^2 + 6 \times 10 + 5 \times 1.$$

Landmark numbers (place value)	10^3	10^2	10^1	1
Number	7	9	6	5

Landmarks in the Invention and Use of Zero

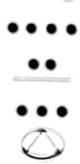
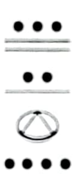
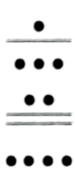
1. The arithmetic properties of the number zero such as adding 0 to any number gives the same number and 0 multiplied by any number gives zero, were used by Aryabhata in his work *'Aryabhatiya'* around 499 CE.
2. The use of 0 as a number at par with any other number, on which one can perform the basic arithmetic operations, was codified by Brahmagupta in his work *Brahma-sphuta-siddhanta* in 628 CE.
3. By introducing 0 as a number, along with the negative numbers, Brahmagupta created a ring, i.e., a set of numbers closed under addition, subtraction and multiplication, thus laying the foundations for modern mathematics.

EXERCISE 4C

1. Represent the following numbers in the Mesopotamian number system.

   (i) 89 (ii) 144 (iii) 302 (iv) 856 (v) 3972

2. What does each of the following numerals represent in the Mayan number system?

   (i)  (ii)  (iii)  (iv) 

3. Represent each of the following numbers using the Mayan system.

   (i) 82 (ii) 105 (iii) 584 (iv) 2043 (v) 16802

4. What does each of the following numerals stand for in the Chinese number system?

   (i)  (ii)  (iii)  (iv) 

5. Represent each of the following numbers in the form of rod numerals.

   (i) 92 (ii) 365 (iii) 2783 (iv) 94621

6. Write each of the following numbers in their expanded form as per the Hindu number system.

   (i) 5283 (ii) 30672 (iii) 89563 (iv) 107086

Things to Remember

1. *A well-defined writing system for expressing numbers by using a standard sequence of digits or symbols that have a fixed order is called a* ***number system****.*
2. *The need for numbers started with the basic needs of counting in daily life.*

3. *The Hindu number system, which is now used as a standard across the world, first originated in India, then spread to the Arab world, and finally reached Europe.*
4. *The most primitive methods of counting involved the use of body parts or marking notches on bones or hard surfaces which are now considered analogous to modern-day tally marks.*
5. *The earliest number systems were based on counting in twos.*
6. *The* ***Roman numerals*** *used combinations of letters from the Latin alphabet to write numbers. The basic Roman numerals were I (for 1), V (for 5), X (for 10), L (for 50), C (for 100), D (for 500) and M (for 1000).*
7. *The symbols representing numbers in a written number system are called* ***numerals.***
8. *In a number system, the numbers that are used as reference points for representing all the numbers in that system are called* ***landmark numbers.***
9. *A number system whose landmark numbers are the powers of a number n is referred to as* ***a base-n number system.***
10. *Number systems with a base that make use of the position of a symbol in determining the landmark number that it is associated with are called* ***positional number systems*** *or* ***place-value systems.***
11. *The Egyptian number system was a base-10 number system but it was not a place-value system.*
12. *The place-value system was observed in the:*
    (i) ***Mesopotamian (Babylonian) number system*** *with base-60 and landmark number 1 ($= 60^0$), 60 ($= 60^1$), 3600 ($= 60^2$), 216000 ($= 60^3$), ... and so on.*
    (ii) ***Mayan number system*** *with base 20 (nearly) and landmark numbers 1, 20 ($= 20 \times 1$), 360 ($= 20 \times 18$), 7200 ($= 20^2 \times 18$), ... and so on.*
    (iii) ***Chinese number system*** *with base 10 and landmark numbers 1 ($= 10^0$), 10 ($= 10^1$), 100 ($= 10^2$), 1000 ($= 10^3$), ... and so on.*
    (iv) ***Hindu number system*** *with base 10 and landmark numbers 1 ($= 10^0$), 10 ($= 10^1$), 100 ($= 10^2$), 1000 ($= 10^3$), ... and so on.*

TEST PAPER 4

1. Represent the following numbers as Roman numerals.

   (i) 367 (ii) 798 (iii) 1454 (iv) 3999

2. Represent the following numbers in the Egyptian system.

   (i) 1023 (ii) 6913

3. Represent the following numbers in the Mayan number system.

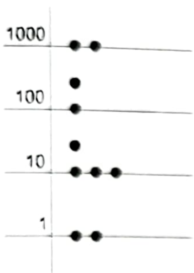
   (i) 249 (ii) 761 (iii) 1126

4. Represent each of the following numbers in the form of rod numerals.

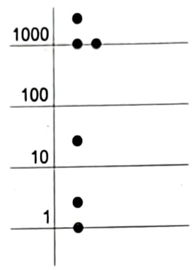
   (i) 683 (ii) 2894 (iii) 5932

5. Write the numerals shown on the abacus below.

   (i)



   (ii)



6. Add the following numerals without converting them to Hindu numerals.

   (i) CCCXIV + CCLXXXVI (ii) DCCXXIX + LXXXVIII

7. Write each of the following numerals in the expanded form as per the Hindu–Arabic number system.

   (i) 8475 (ii) 59651 (iii) 643281

Mark (✓) against the correct answer in each of the following.

8. Which of the following is a valid representation in Roman numeral system?

   (a) VX (b) XXXX (c) LL (d) XC

9. Which of the following represents 10^6 in the Egyptian number system?

   (a)  (b)  (c)  (d) 

10. Which number cannot be represented in a base-5 number system?

    (a) 0 (b) 1 (c) 2 (d) All the above

11. The landmark numbers of a base-7 number system are

    (a) $7, 7 \times 1, 7 \times 2, 7 \times 3, \ldots$ (b) $7^0, 7^1, 7^2, 7^3, \ldots$

    (c) $7, 7 + 1, 7 + 2, 7 + 3, \ldots$ (d) $7, 7 \times 2, 7 \times 2^2, 7 \times 2^3, \ldots$

12. In which of the following number systems was a symbol for zero introduced?

    (a) Egyptian (b) Mesopotamian (c) Mayan (d) Chinese

13. Which of the following was not a base-10 number system?

    (a) Chinese (b) Mayan (c) Egyptian (d) Hindu-Arabic

14. The landmark numbers in the Mesopotamian number system were

(a) 1, 60, 120, 180, 240, ...

(b) 1, 60, 600, 6000, 60000, ...

(c) 1, 60, 3600, 21600, ...

(d) 1, 60, 300, 6000, 30000, ...

15. ***Fill in the blanks.***

(i) The symbol used for 1000 in the Roman number system is ....

(ii) The Chinese rod numerals consisted of two different sets of symbols for digits 1 to 9, called the ...... and the ....

(iii) The earliest number systems were based on counting in ....

(iv) The Roman numerals used ...... alphabets to write numbers.

(v) The symbol for 10^2 in Egyptian number system is ....

16. ***State whether each of the following statements is true or false.***

(i) The Egyptian number system is a base-10 number system.

(ii) The Chinese rod numeral system used a blank space for zero.

(iii) The Mesopotamian number system is a base-20 number system.

(iv) In the Mayan number system, a bar represented zero.

❑